Discrete Distributions — Computational Examples

Binomial Distribution

Suppose $X \sim \text{Binomial}(n=50,\ p=1/3)$

(I) What is $P(X=10)$?

$$P(X=10) = \binom{50}{10}(1/3)^{10}\cdot(2/3)^{40}$$

In R this is given by

`dbinom(10, 50, 1/3)`

(x, n, p)

(II) What is $P(X \le 10)$?

The formula for this is terrible, so let's just use R

`pbinom(10, 50, 1/3)`

[Recall: dbinom is pmf $f(x) = P(X=x)$

pbinom is cdf $F(x) = P(X \le x)$]

(III) What is $P(X<10)$?

To use pbinom we must write this in terms of $P(X \le k)$

$$P(X<10) = P(X \le 9)$$

Because X can only be integer

`pbinom(9, 50, 1/3)`

(IV) What is $P(X>20)$?

To use pbinom we must write this in terms of $P(X \le k)$

$$P(X>20) = P(X \text{ not} \le 20)$$

$$= 1 - P(X \le 20)$$

`1 - pbinom(20, 50, 1/3)`

(V) What is $P(10<X<20)$

$$P(10<X<20) = P(X<20) - P(X \le 10)$$

`pbinom(19, 50, 1/3) - pbinom(10, 50, 1/3)`

Note: For binomial random variables

$$P(a < X \le b) = F(b) - F(a)$$

"pbinom(b,..) − pbinom(a,..)"

This should remind you of Fund. Thm. of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

area for $a \le x \le b$

(continuing example problems for $X \sim \text{Binom}(50, 1/3)$...)

(VI) Find x so that $P(X \le x) \approx 1/2$.

We need to invert the cdf $F(x)$

In R, this is given by the command

qbinom(...) ← "q" for "quantile"

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qbinom( 1/2, 50, 1/3 )
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(VII) Find x so that $P(10 < X \le x) \approx 1/4$.

$$P(10 < X \le x) = P(X \le x) - P(X \le 10)$$

Want

$$1/4 \approx P(X \le x) - \text{pbinom}(10, 50, 1/3)$$

$$P(X \le x) \approx 1/4 + \text{pbinom}(10, 50, 1/3)$$

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qbinom( 1/4 + pbinom(10, 50, 1/3), 50, 1/3 )
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(VIII) Find x so that $P(X < x) \approx 1/2$.

NOT ≤ !!

$$P(X < x) = P(X \le x-1)$$ ← Because X can only be integer

Thus

$$\text{qbinom}(1/2, 50, 1/3) = x - 1 \quad !!$$

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qbinom( 1/2, 50, 1/3 ) + 1
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(IX) Find x so that $P(X > x) \approx 1/4$.

If $P(X > x) = 1/4$

then $P(X \le x) = 1 - 1/4 = 3/4$

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qbinom( 3/4, 50, 1/3 )
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[For other distributions the computations are similar:]

EX If $X \sim$ Neg. Binom. $(4, 1/10)$ then what is

$$P(30 \le X \le 40) \; ?$$

$$P(30 \le X \le 40) = P(X \le 40) - P(X < 30)$$
$$= P(X \le 40) - P(X \le 29)$$

`pnbinom(40, 4, 1/10) - pnbinom(29, 4, 1/10)`

EX If $X \sim$ Geometric $(1/5)$ then find x so that

$$P(x < X < 10) \approx 1/3 .$$

$$1/3 \approx P(x < X < 10) = P(X < 10) - P(X \le x)$$
$$= P(X \le 9) - P(X \le x)$$
$$P(X \le x) \approx P(X \le 9) - 1/3$$

`qgeom( pgeom(9, 1/5) - 1/3 , 1/5)`

[**Note:** It is possible to ask questions like "Find x so $P(a < X < x) = p$" which have no answer:

2) No solution if $p - P(X \le a)$ is not between 0 & 1

This should be a probability]

Word Problem Example

Suppose $1/10$ of all soda bottles have a special prize. What is the probability of getting at least 4 prizes from 20 sodas?

Let X = #prizes from 20 sodas.

Then $X \sim$ Binomial $(20, 1/10)$

We want $P(X \ge 4)$

$$= 1 - P(X \le 3)$$

`1 - pbinom(3, 20, 1/10)`